

# Intersection and Rotation of Assumption Literals Boosts Bug-Finding

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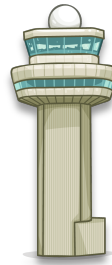
# Design-Space Exploration

# Design-Space Exploration

What is a design-space?

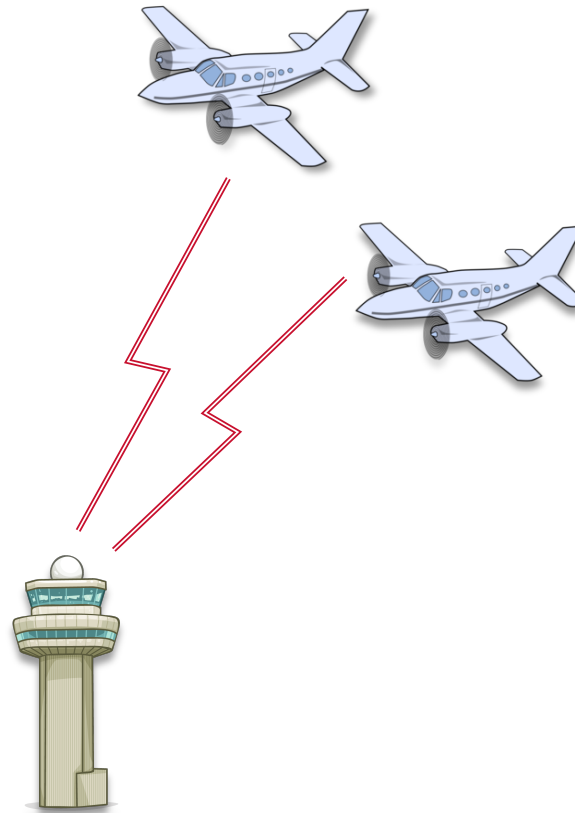
# Design Space

## Airspace Allocation



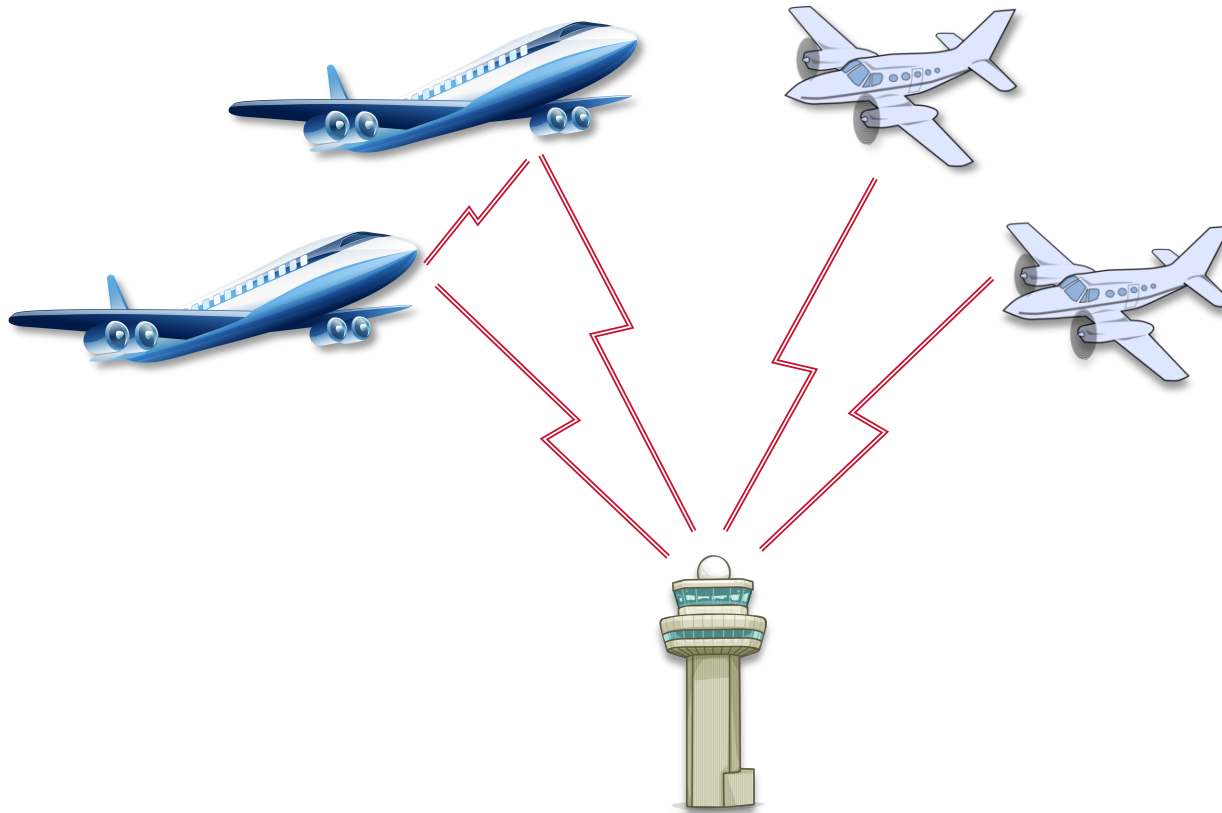
# Design Space

## Airspace Allocation



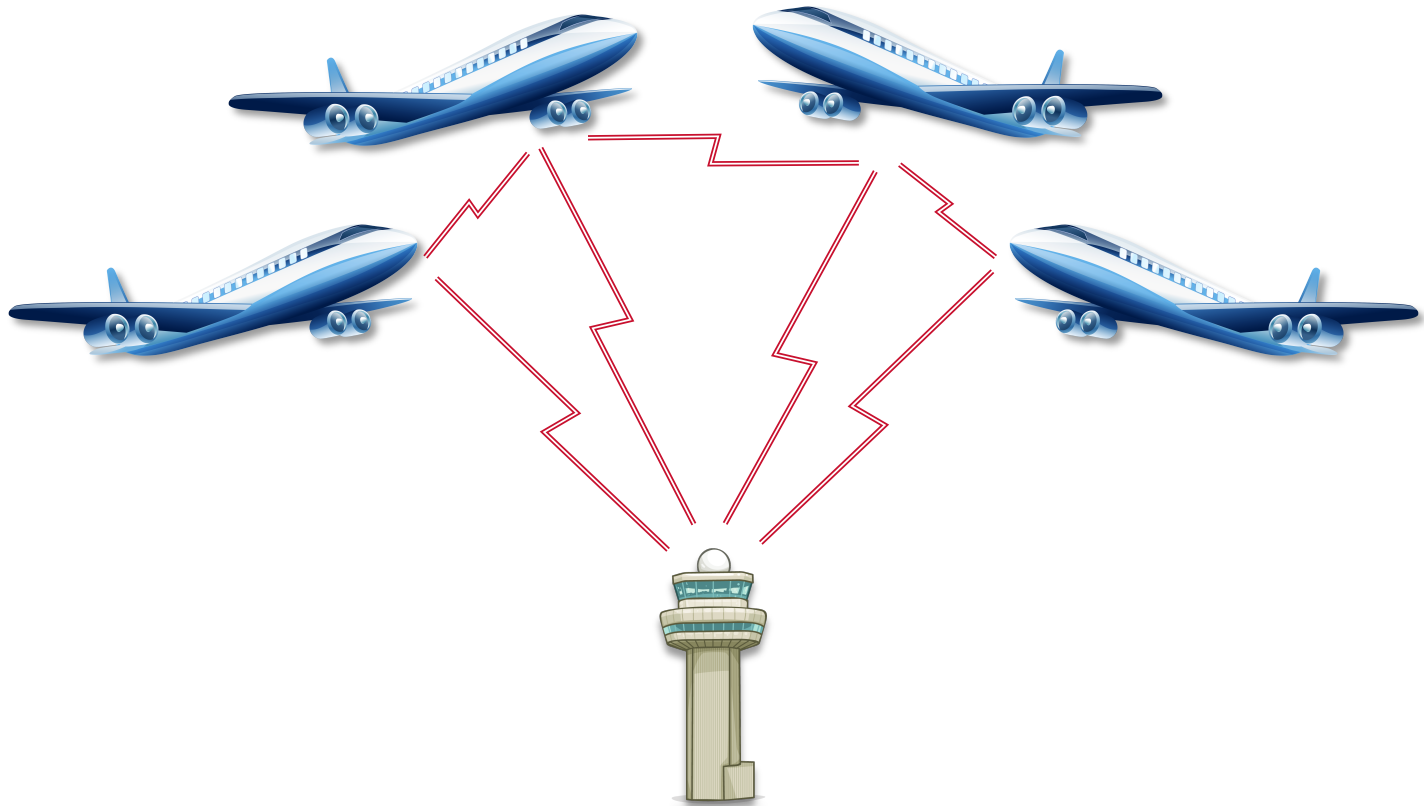
# Design Space

## Airspace Allocation



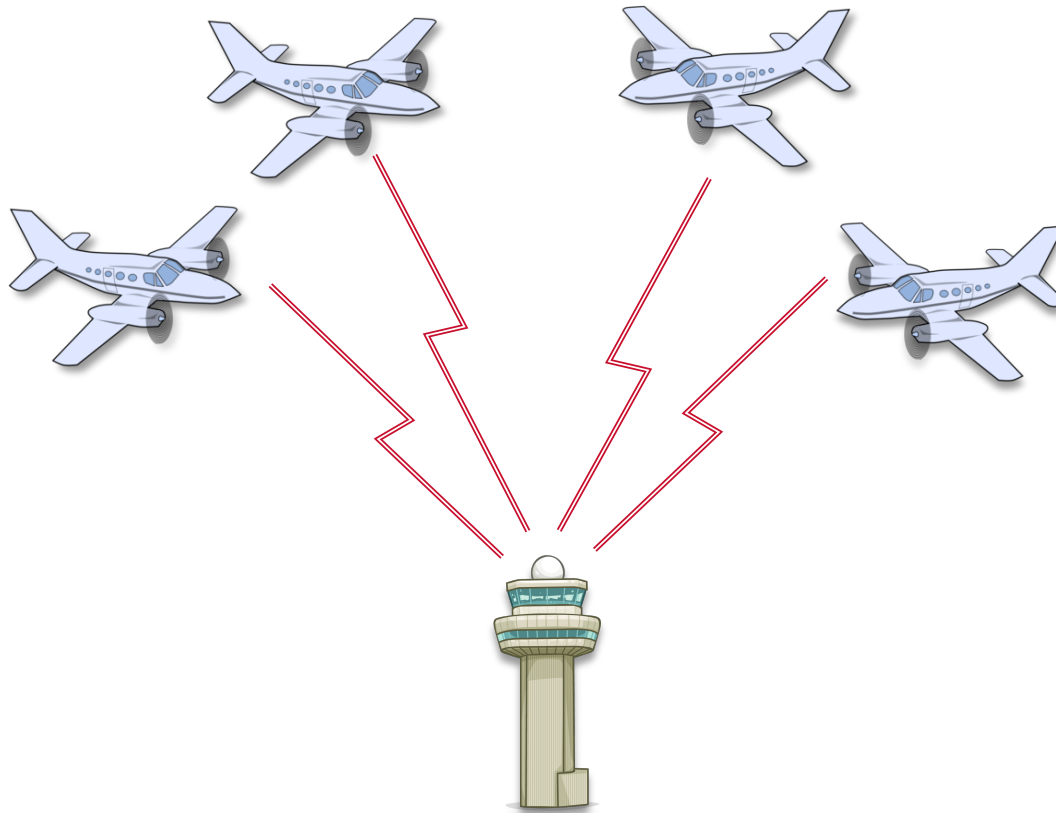
# Design Space

## Airspace Allocation



# Design Space

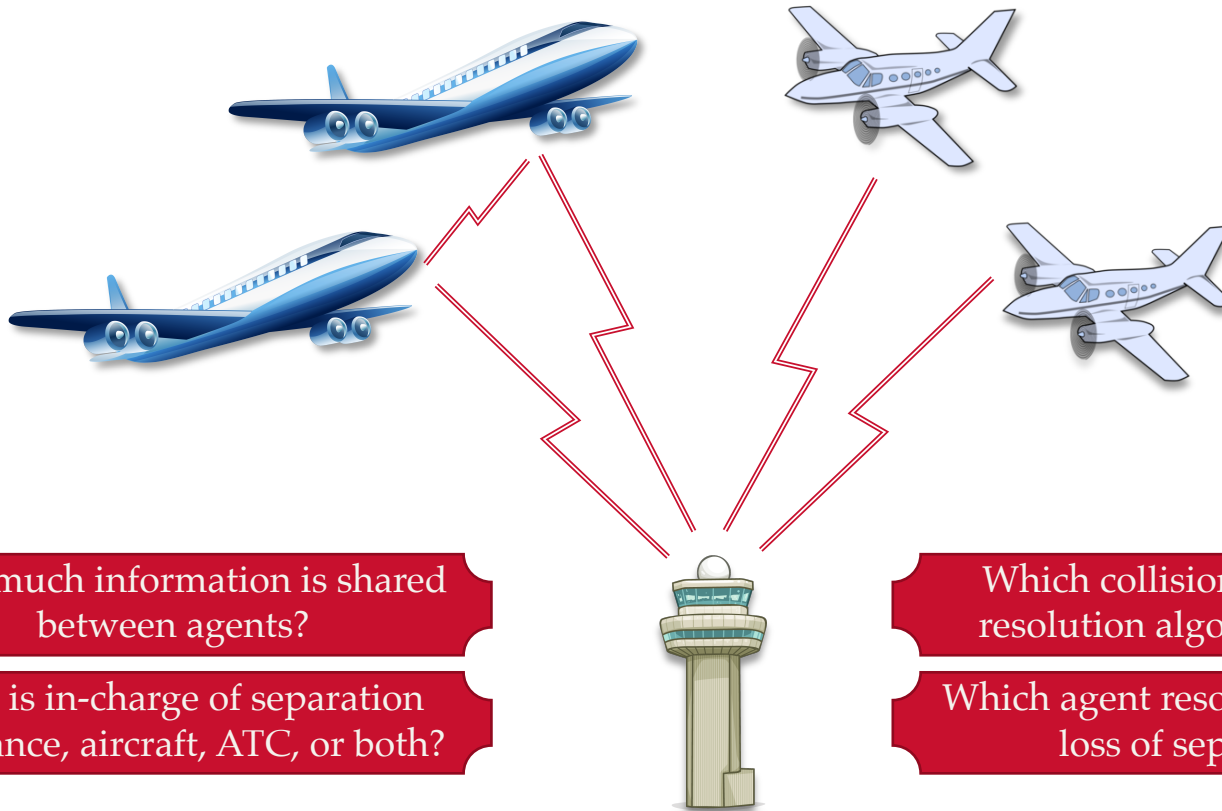
## Airspace Allocation





# Design Space

## Airspace Allocation



How much information is shared between agents?

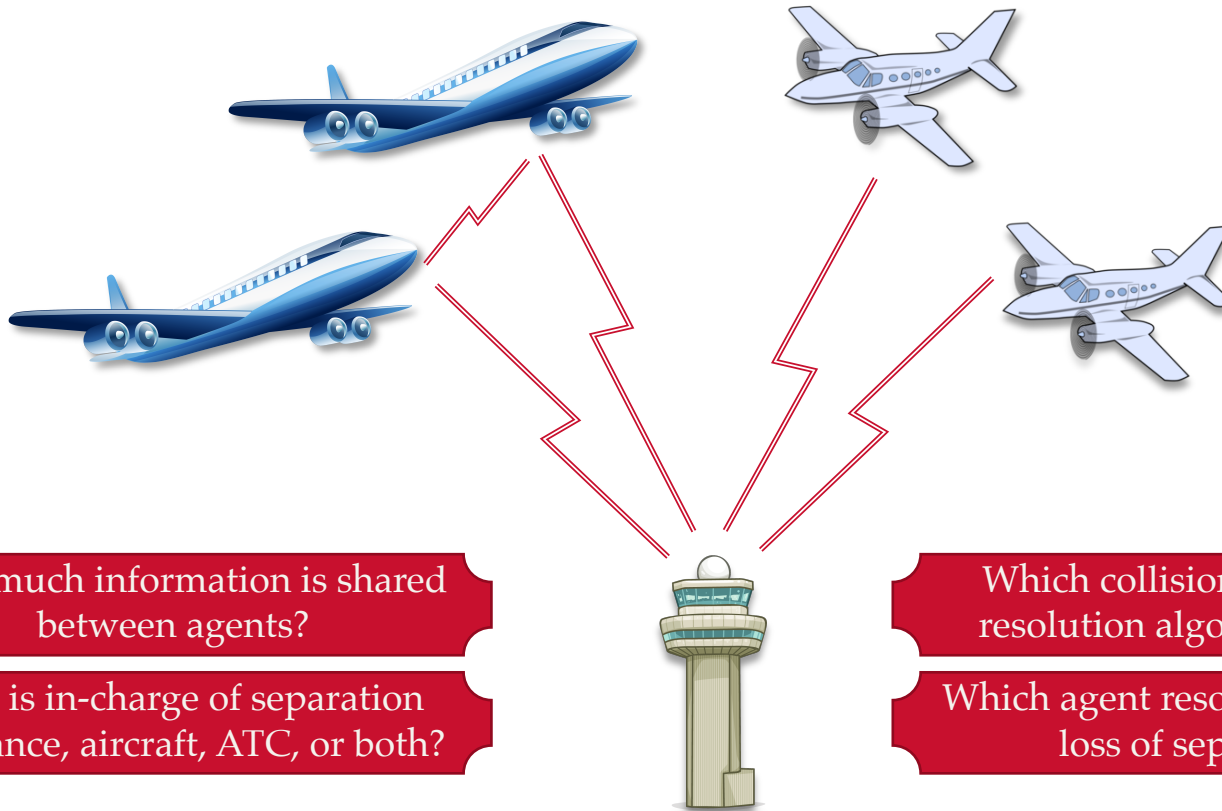
Who is in-charge of separation assurance, aircraft, ATC, or both?

Which collision detection & resolution algorithm to use?

Which agent resolves a potential loss of separation?

# Design Space

## Airspace Allocation



Lots of Design Choices!

# Design-Space Exploration

What is a design-space?

# Design-Space Exploration

What is a design-space?

Set of possible design choices for a system.

# Design-Space Exploration

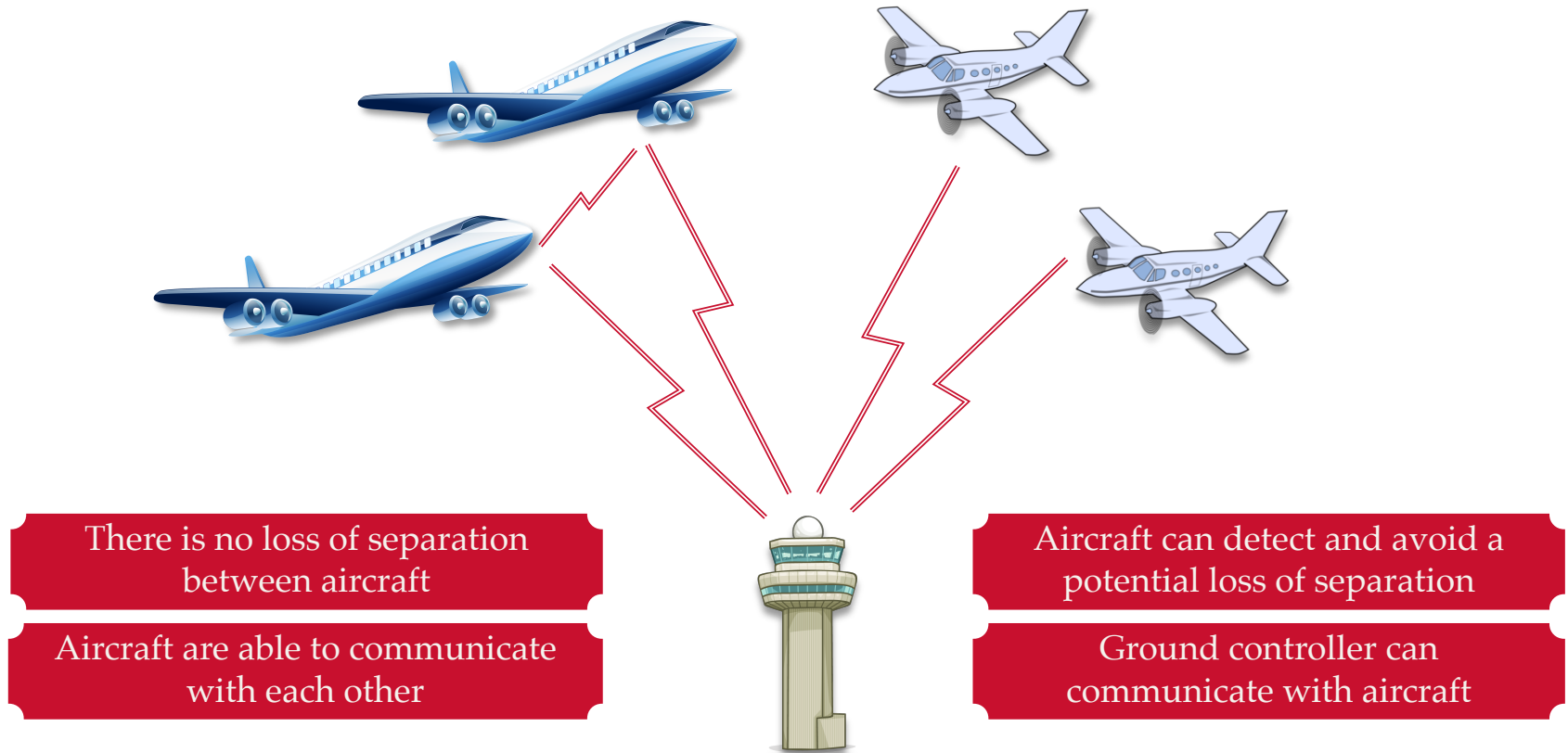
What is a design-space?

Set of possible design choices for a system.

What is a design-space exploration?

# Design-Space Exploration

## Airspace Allocation



Find design choices that satisfy requirements

# Design-Space Exploration

What is a design space?

Set of possible design choices for a system.

What is a design-space exploration?

# Design-Space Exploration

## What is a design space?

Set of possible design choices for a system.

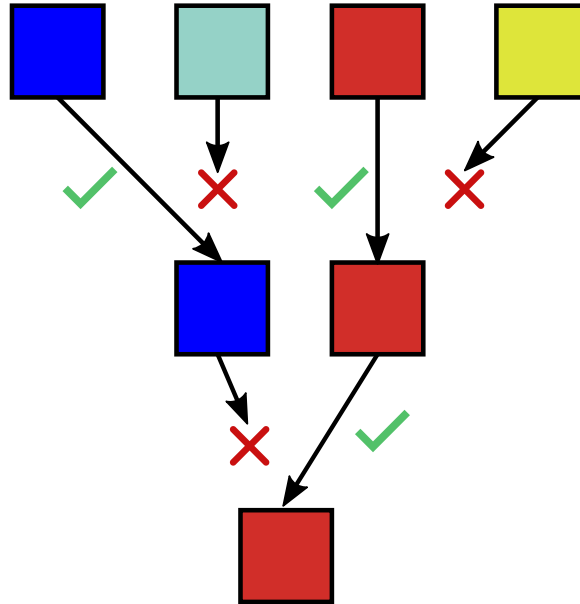
## What is a design-space exploration?

Design-time analysis to evaluate design choices exhaustively.



# Design-Space Exploration

Complex systems are modeled as design spaces.



Alternative comparison via design space exploration

Model Checking!

# Model Checking Design Spaces

Model Set

$$\mathcal{M} = \{M_1, \dots, M_n\}$$

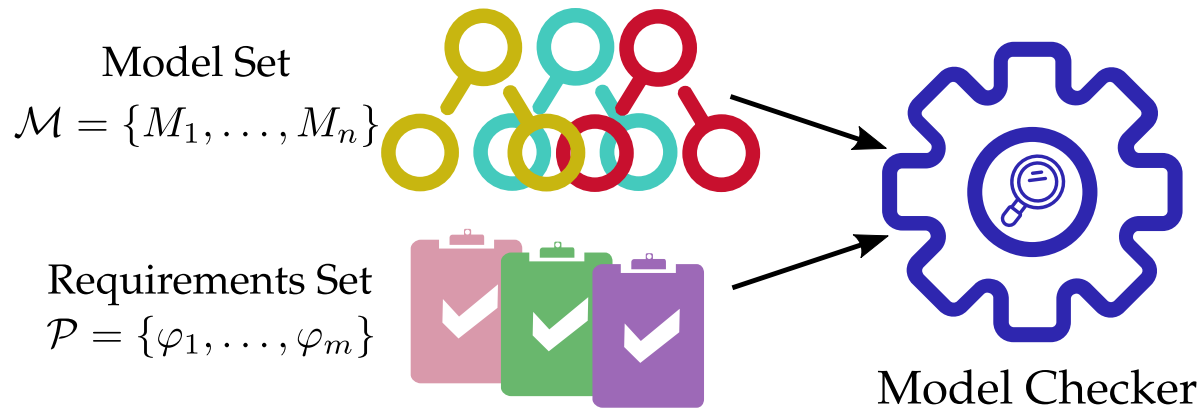


Requirements Set

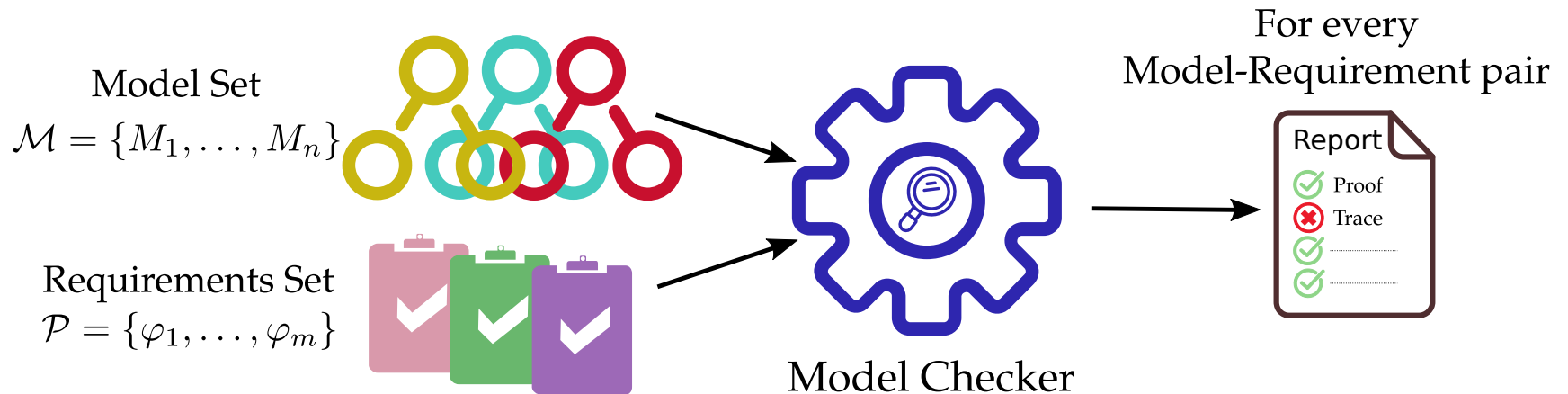
$$\mathcal{P} = \{\varphi_1, \dots, \varphi_m\}$$



# Model Checking Design Spaces



# Model Checking Design Spaces



Design-space model checking entails  
**multi-model/requirement checking**

## Our Goal

Make model-checking for design-spaces more scalable

# Multi-model/requirement Checking

Design-space  
reduction

Incremental  
Verification

Improved  
Orchestration

Model-checking  
algorithms

# Multi-model/requirement Checking

Design-space  
reduction

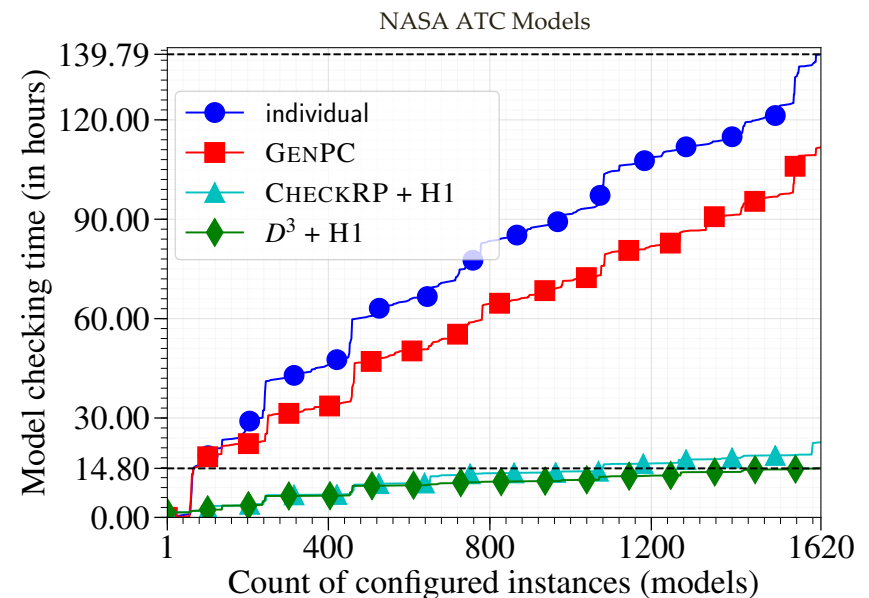
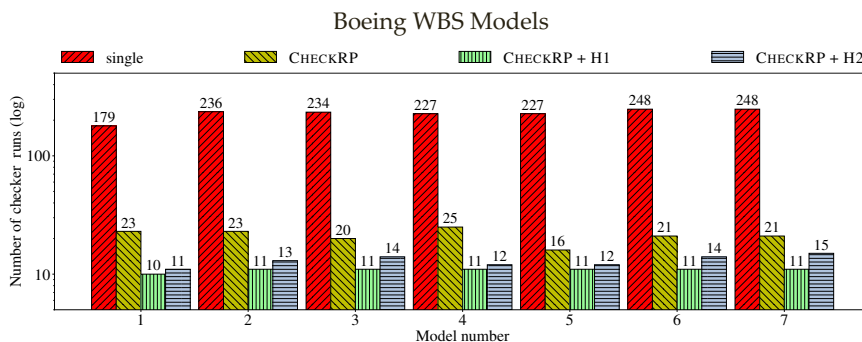
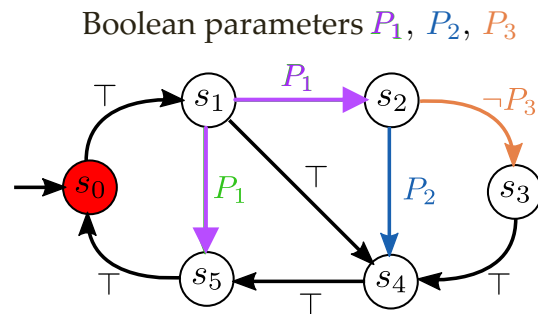
Incremental  
Verification

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# Design-space Reduction<sup>1</sup>

- Generate design-space models from a meta-model
  - Combinatorial transitions systems (CTS), behavior enabled by parameters
- D<sup>3</sup> algorithm to reduce number of model-property pairs
  1. Finding redundant models, or models with exact same behavior (GenPC)
  2. Reducing number of requirements by finding logical dependencies (CheckRP)



Upto 9.0x speedup

# Multi-model/requirement Checking

Design-space  
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# Multi-model/requirement Checking

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Verification

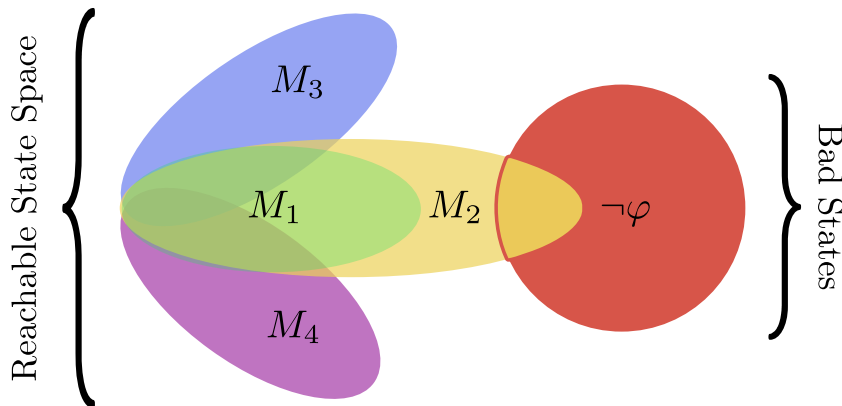
Improved  
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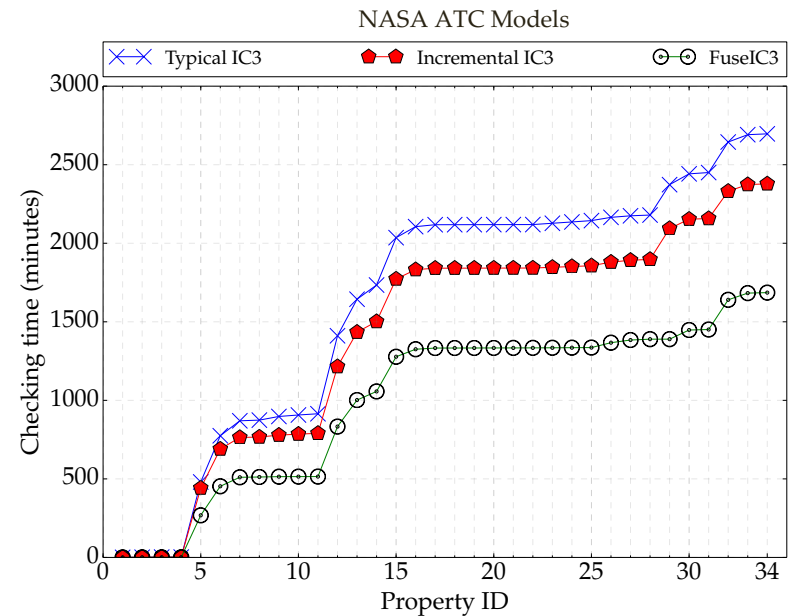
# Incremental Verification<sup>2,3</sup>

- The different design-space models have overlapping state spaces
  - Generated from the same meta-model, overlapping behavior
- FuseIC3 algorithm reuses reachable state approximations
  1. IC3 frames are stored and “repaired” across multiple model-checking runs<sup>2</sup>
  2. Very fast verification when model-delta is small, regressions runs<sup>3</sup>

Set of related models  $\{M_1, M_2, M_3, M_4\}$   
Safety property  $\varphi$



1. Check  $M_1$  with  $\varphi \rightarrow M_1 \models \varphi$
2. Check  $M_2$  with  $\varphi \rightarrow M_2 \not\models \varphi$



Upto 5.48x speedup

<sup>2</sup> R. Dureja and K. Y. Rozier. “FuseIC3: An Algorithm for Checking Large Design Spaces” (FMCAD 2017)

<sup>3</sup> R. Dureja and K. Y. Rozier. “Incremental Design-Space Model Checking via Reusable Reachable State Approximations.” (under submission)

# Multi-model/requirement Checking

Design-space  
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Incremental  
Verification

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# Multi-model/requirement Checking

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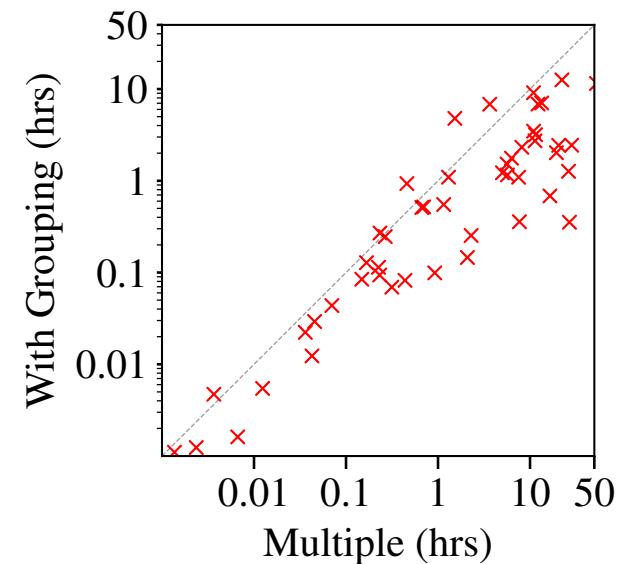
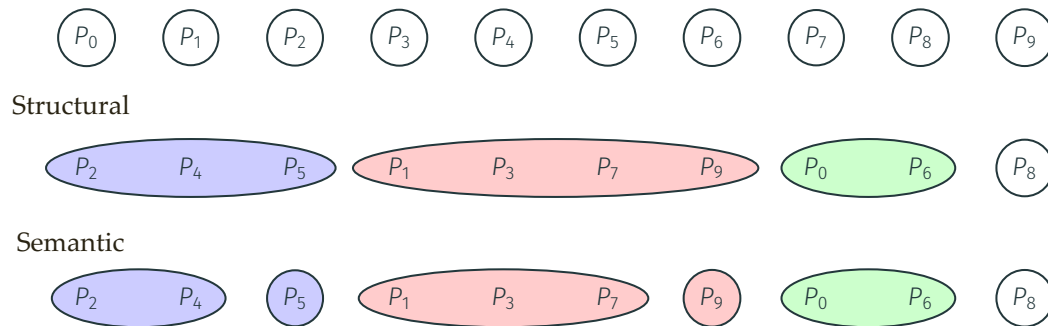
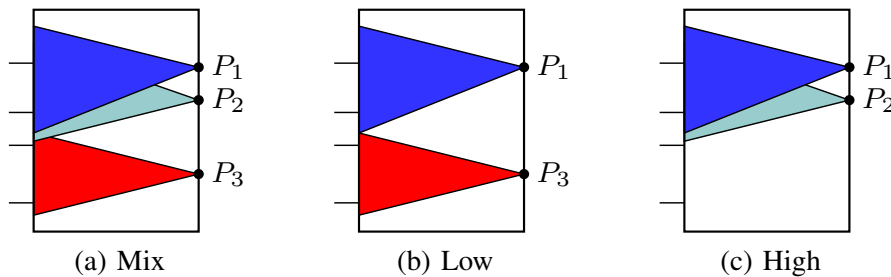
Incremental  
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# Improved Orchestration<sup>4</sup>

- Partially-order models/requirements to maximize reuse
  - Requirement grouping based on COI (structural and semantic)
- Improved localization abstraction
  - Semantically similar requirements are localized concurrently



Upto 72x speedup

# Multi-model/requirement Checking

Design-space  
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# Multi-model/requirement Checking

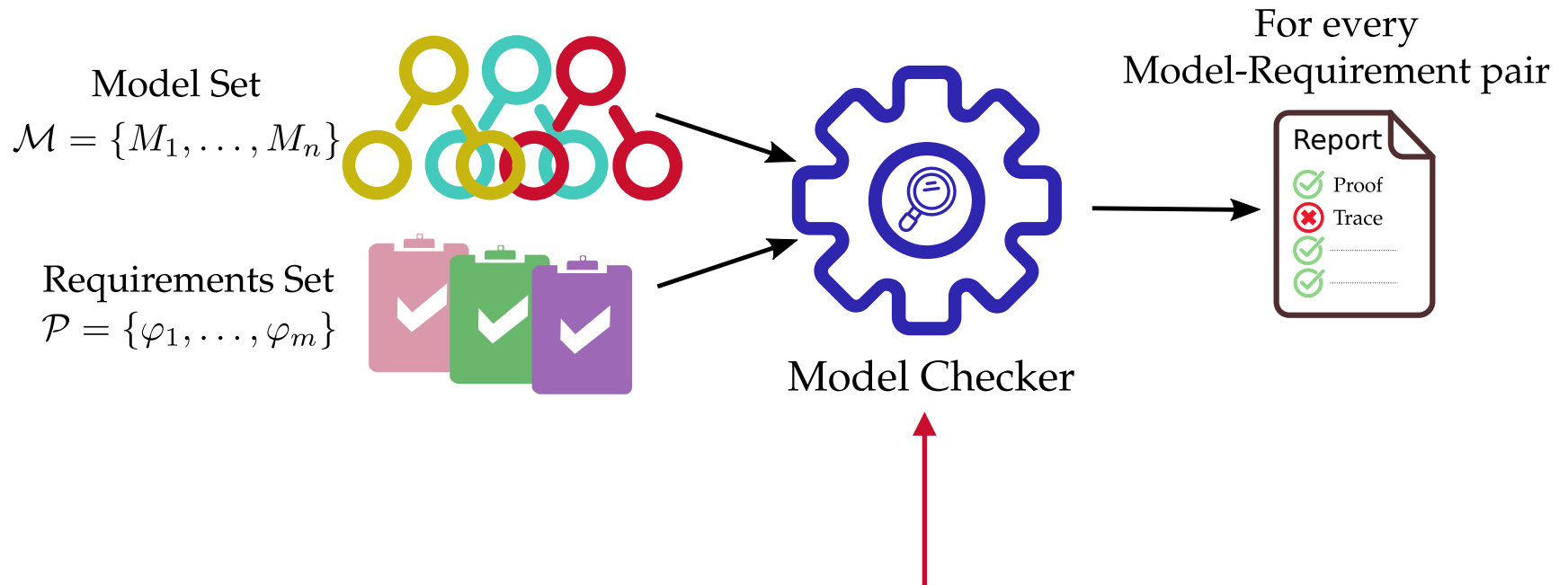
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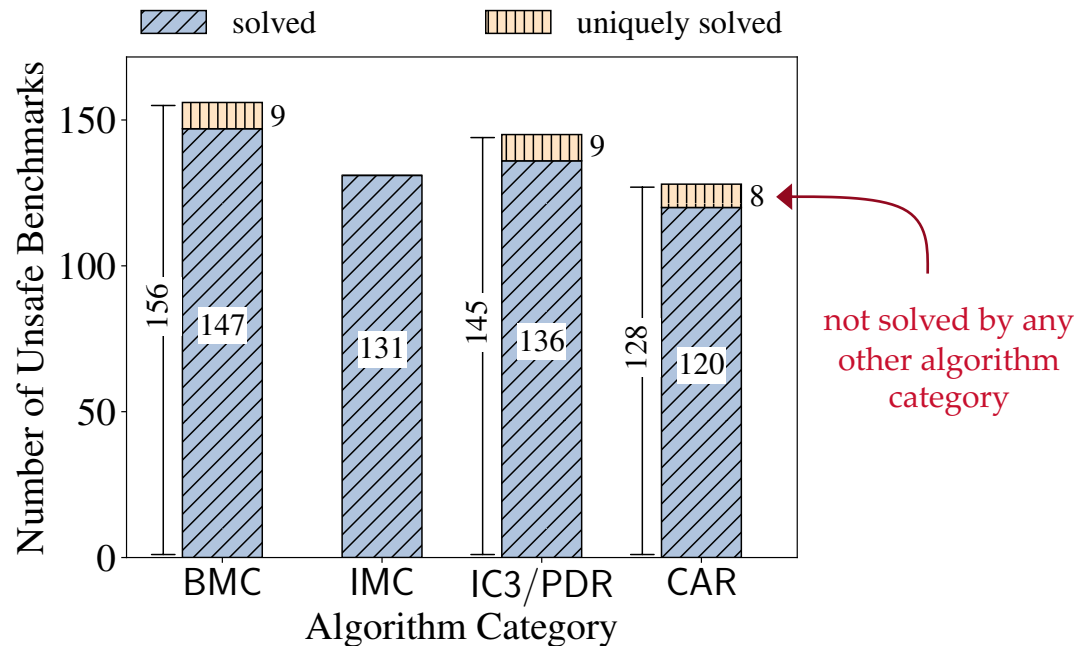
# Model Checking Algorithms





# Model Checking Algorithms<sup>6,7</sup>

- Improve SAT-based model checking algorithms
  - Complementary approximate reachability (CAR) as proof-of-concept<sup>5</sup>
- Heuristics to improve bug-finding performance of CAR
  - SimpleCAR can find bugs not found by IC3/BMC<sup>6</sup>; slow convergence
  - Better SAT-query to improve performance of SimpleCAR<sup>7</sup>
- Also applicable to IC3; more scalable design-space checking



<sup>5</sup> J. Li, S. Zhu, Y. Zhang, G. Pu, and M. Y. Vardi. "Safety model checking with complementary approximations" ICCAD (2017)

<sup>6</sup> J. Li, R. Dureja, G. Pu, K. Y. Rozier, M. Y. Vardi. "SimpleCAR: An Efficient Bug-Finding Tool Based on Approximate Reachability" (CAV 2018)

<sup>7</sup> R. Dureja, J. Li, G. Pu, M. Y. Vardi, K. Y. Rozier. "Intersection and Rotation of Assumption Literals Boosts Bug-Finding" (VSTTE 2019)

# Standard Reachability Analysis

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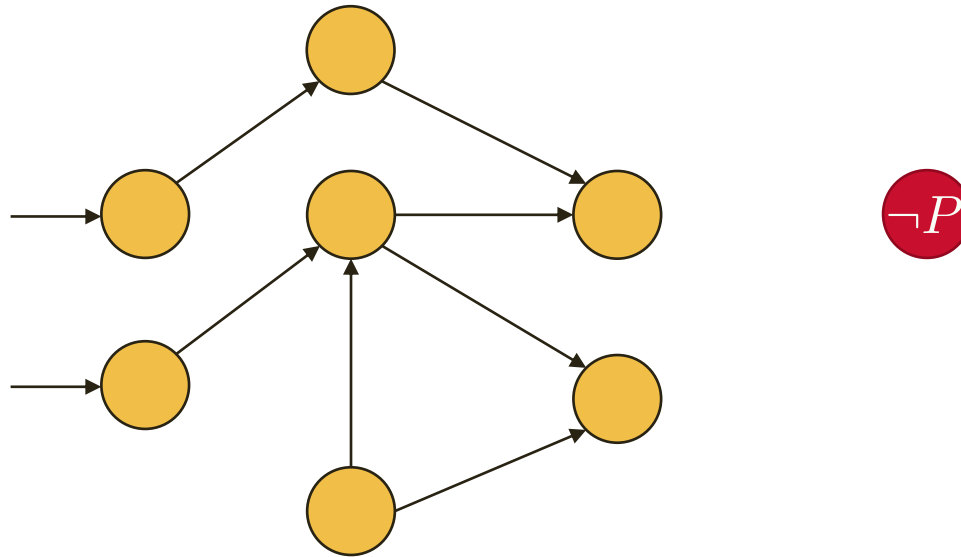
Model  $M = (V, I, T)$

Safety Property  $P$

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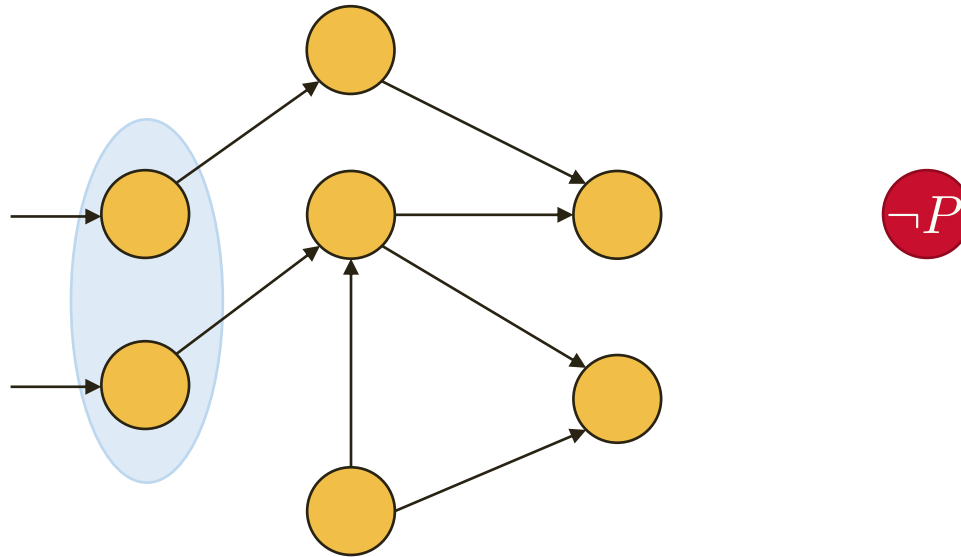
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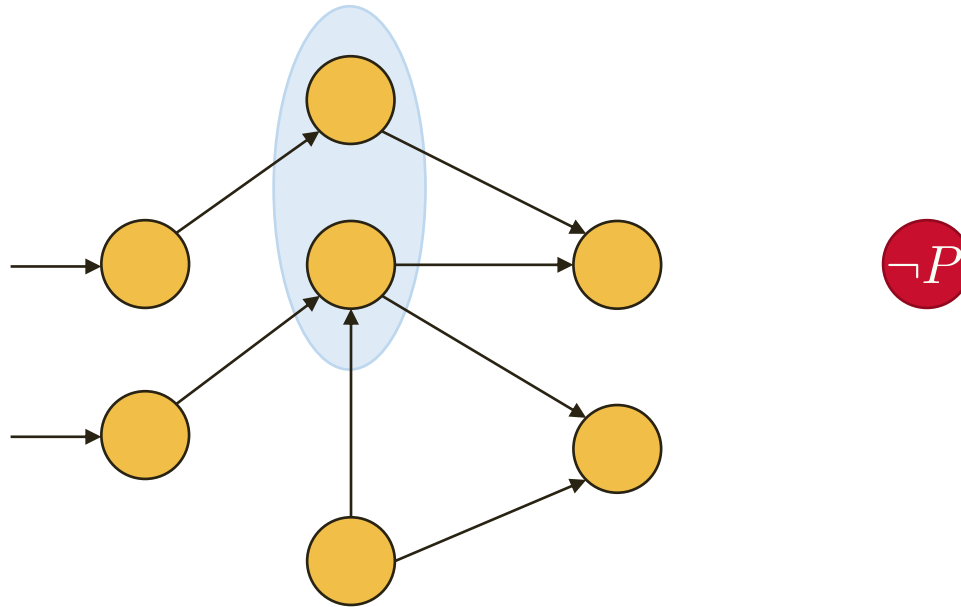
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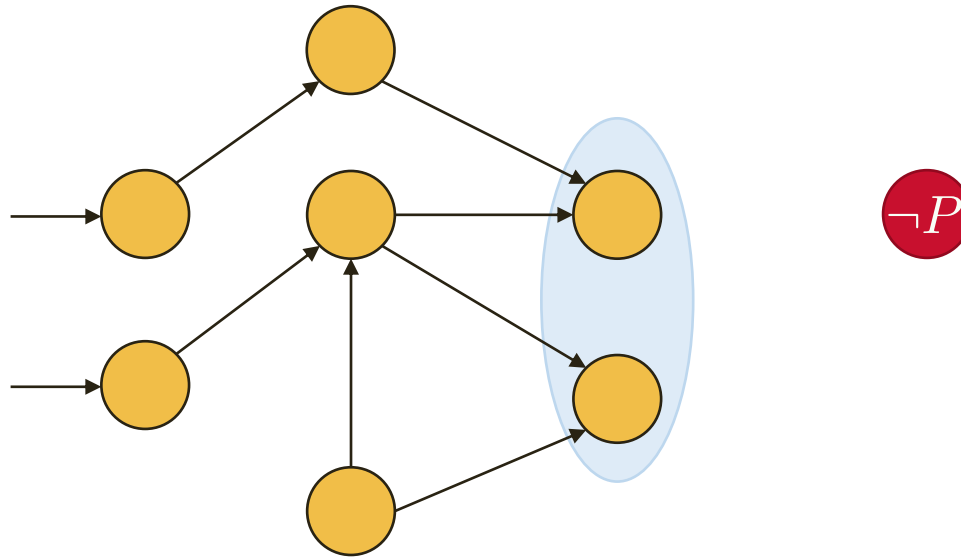
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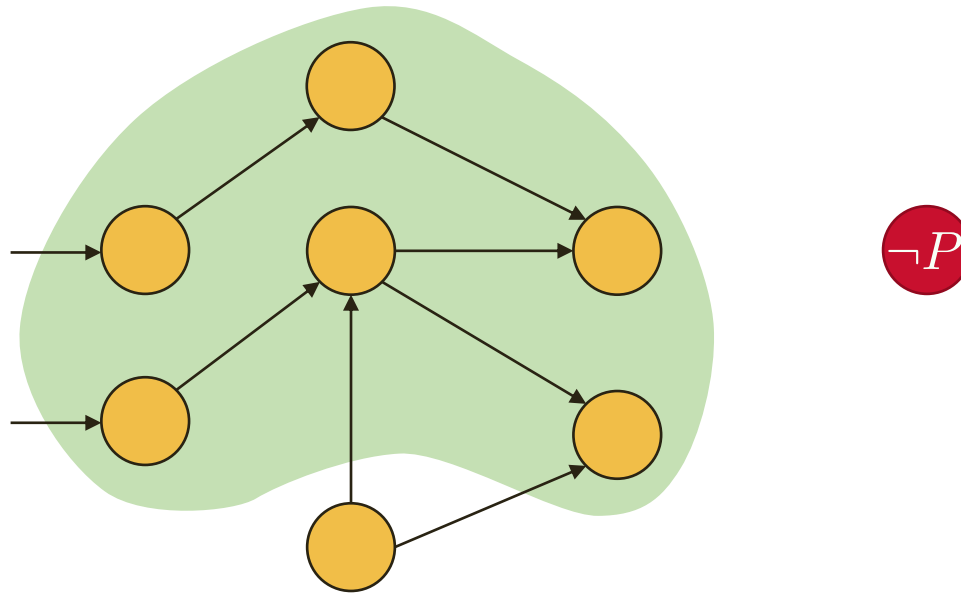
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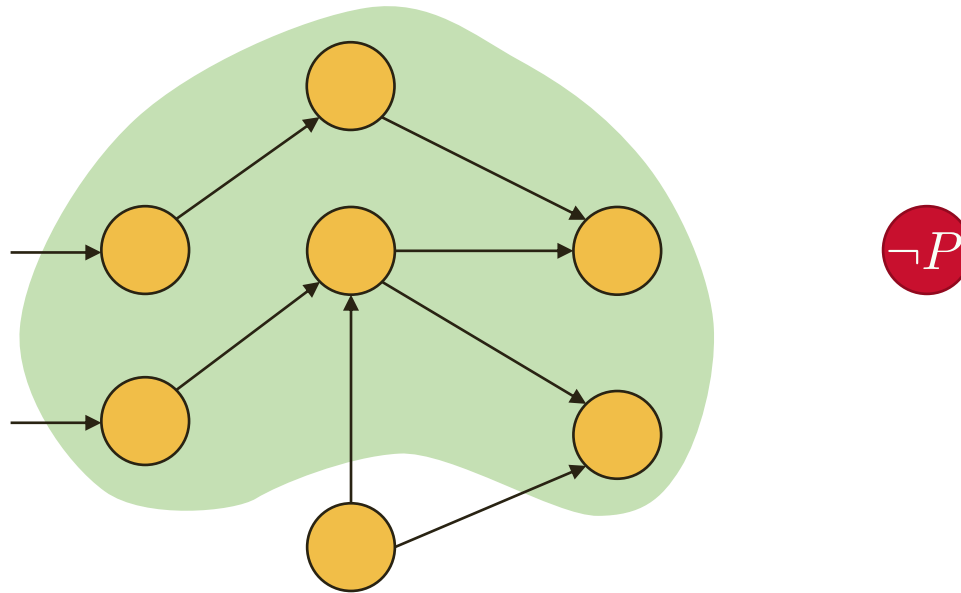




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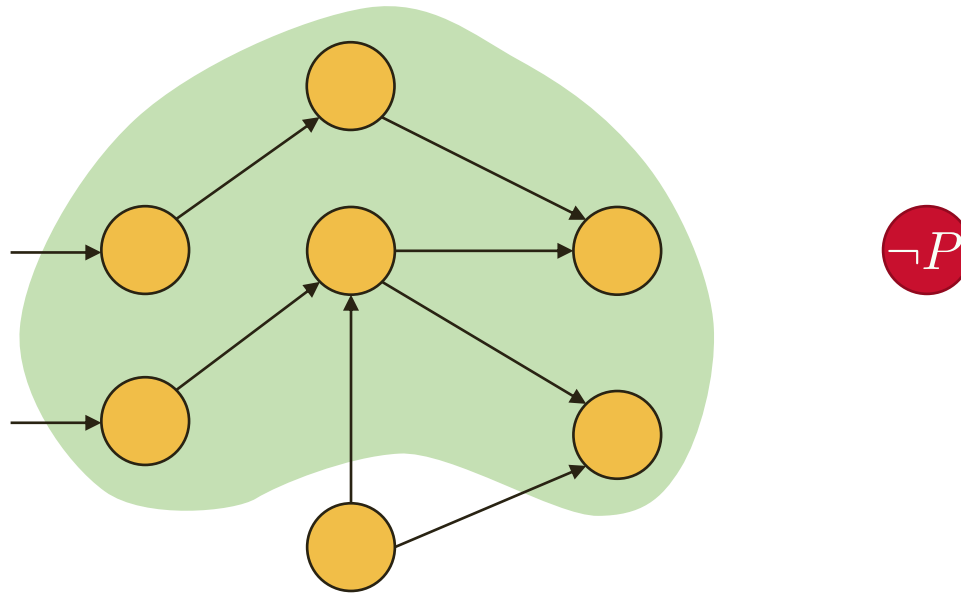


$M \models P$

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Model  $M = (V, I, T)$

Safety Property  $P$



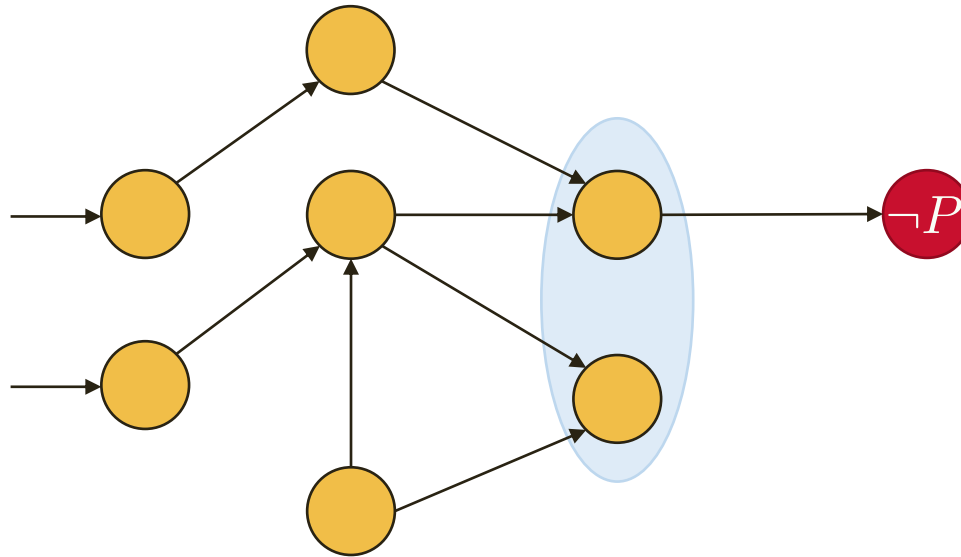
$$M \models P$$

$M$  is **safe** with respect to  $P$

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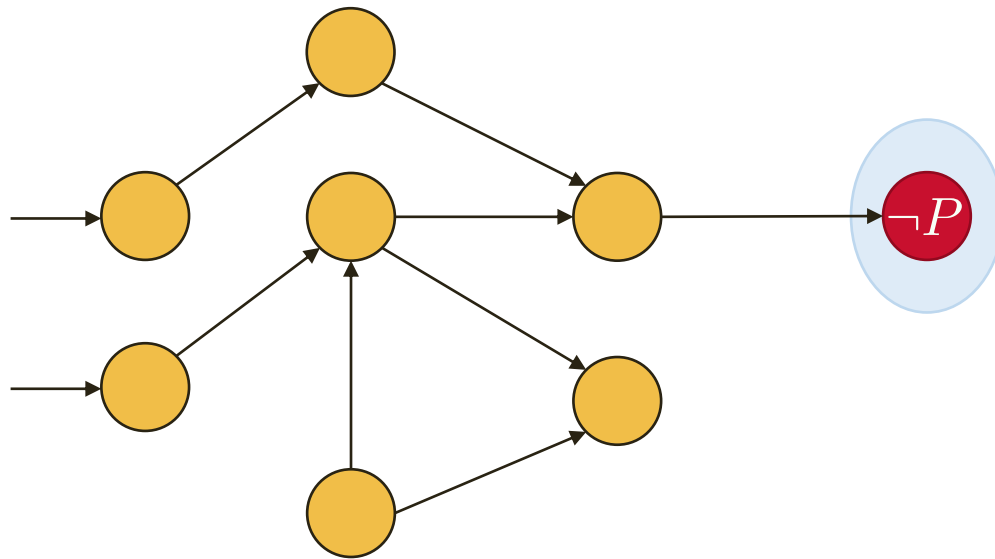
Safety Property  $P$



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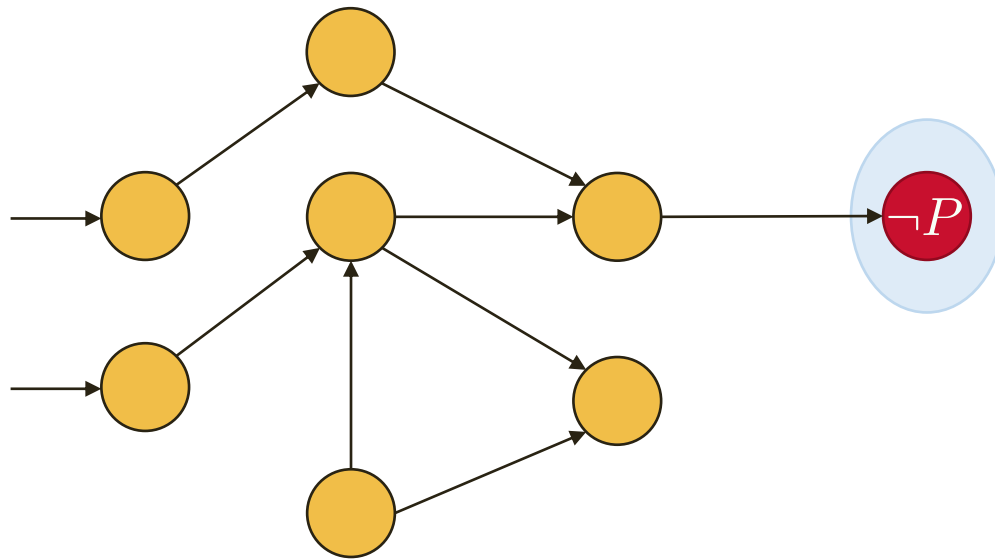
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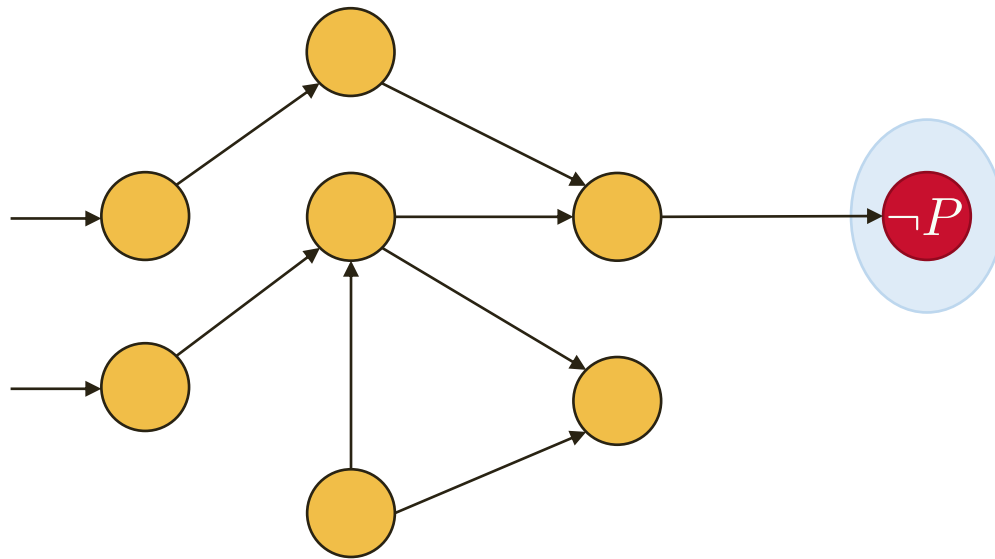


$M \not\models P$

# Standard Reachability Analysis

Model  $M = (V, I, T)$

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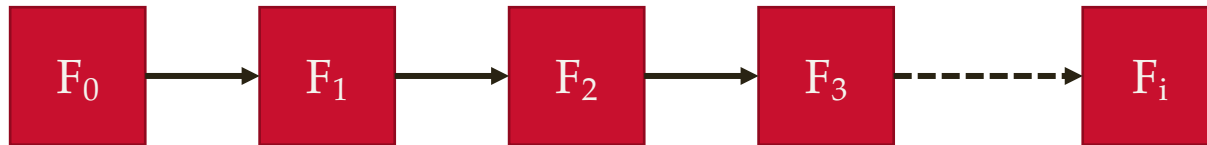


$$M \not\models P$$

M is **unsafe** with respect to P

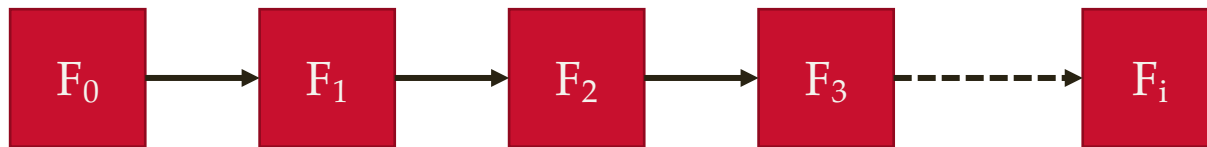
# Complementary Approximate Reachability

Standard Reachability Analysis



# Complementary Approximate Reachability

## Standard Reachability Analysis

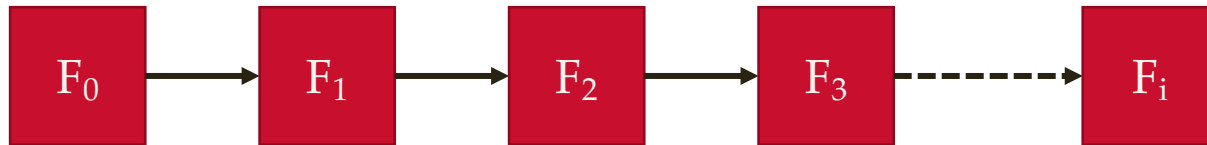


Basic:  $F_0 = I$



# Complementary Approximate Reachability

## Standard Reachability Analysis

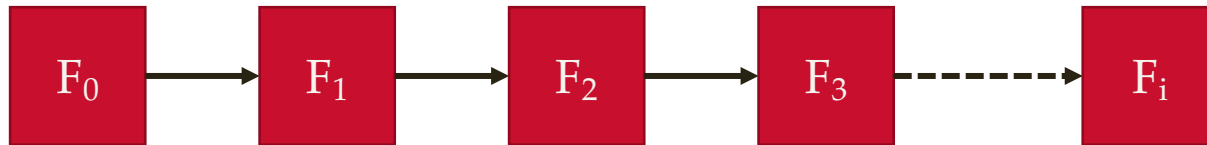


Basic:  $F_0 = I$

Induction:  $F_{i+1} = \text{Reach}(F_i)$

# Complementary Approximate Reachability

## Standard Reachability Analysis



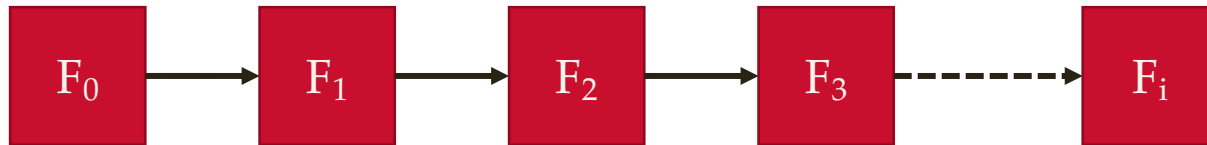
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Terminate:  $F_{i+1} \subseteq \bigcup_{0 \leq j \leq i} F_j$

# Complementary Approximate Reachability

## Standard Reachability Analysis



Basic:  $F_0 = I$

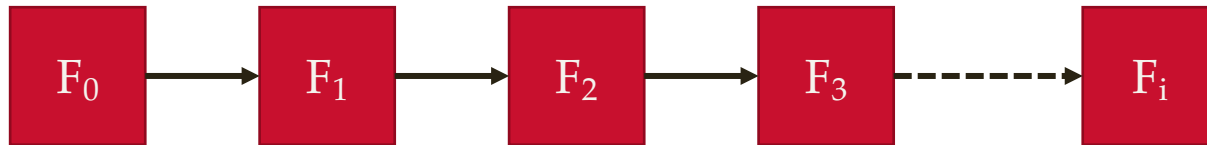
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Check:  $F_i \cap \neg P \neq \emptyset$

# Complementary Approximate Reachability

## Standard Reachability Analysis



Basic:  $F_0 = I$

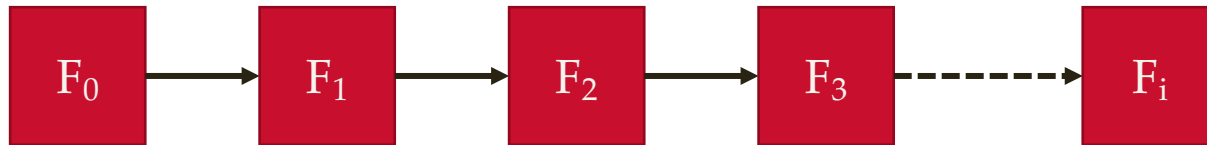
Induction:  $F_{i+1} = \text{Reach}(F_i)$

Terminate:  $F_{i+1} \subseteq \bigcup_{0 \leq j \leq i} F_j$  ← Safety

Check:  $F_i \cap \neg P \neq \emptyset$  ← Unsafety  
(bug-finding)

# Complementary Approximate Reachability

## Standard Reachability Analysis



Basic:  $F_0 = I$

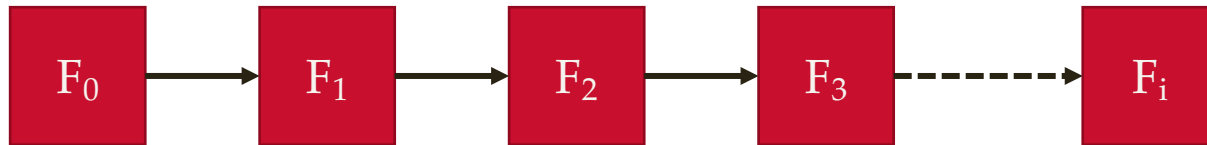
Induction:  $F_{i+1} \stackrel{\circ}{=} \text{Reach}(F_i)$

Terminate:  $F_{i+1} \subseteq \bigcup_{0 \leq j \leq i} F_j \longleftarrow$  Safety

Check:  $F_i \cap \neg P \neq \emptyset \longleftarrow$  Unsafety  
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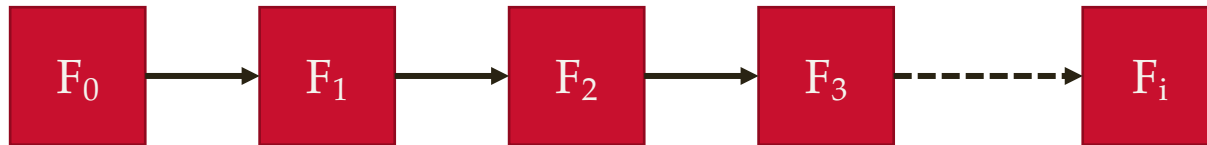
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**Maintaining exact frame sequences is hard; more states in memory**

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**Maintaining exact frame sequences is hard; more states in memory**

CAR uses approximate sequences

# Complementary Approximate Reachability

Maintains two approximate sequences



# Complementary Approximate Reachability

Maintains two approximate sequences

Forward Sequence



# Complementary Approximate Reachability

Maintains two approximate sequences

Forward Sequence  
(over-approximate)



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# Complementary Approximate Reachability

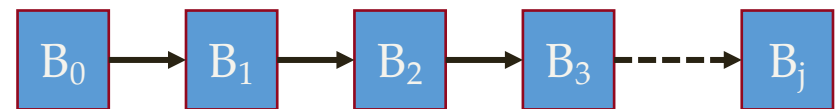
Maintains two approximate sequences

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Backward Sequence  
(under-approximate)



Basic:  $B_0 = \neg P$   
 Induction:  $B_{j+1} \subseteq \text{Reach}^{-1}(B_j)$   
 Check:  $B_j \cap I \neq \emptyset$

Inverse transition

# Complementary Approximate Reachability

Maintains two approximate sequences

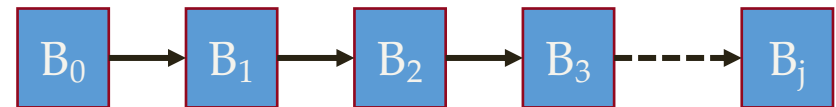
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Safety Checking

Backward Sequence  
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Unsafety Checking

# Complementary Approximate Reachability

Maintains two approximate sequences

## Forward-CAR

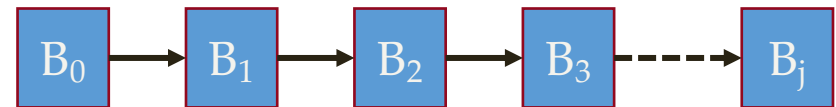
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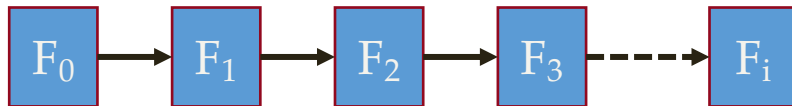
Unsafety Checking

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Maintains two approximate sequences

## Backward-CAR

Forward Sequence



Backward Sequence

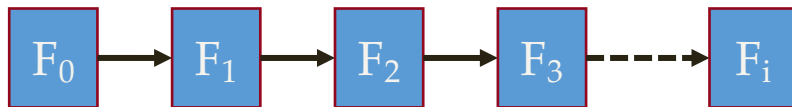


# Complementary Approximate Reachability

Maintains two approximate sequences

## Backward-CAR

Forward Sequence  
(under-approximate)



Backward Sequence



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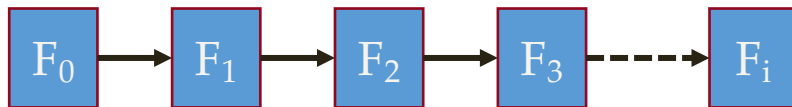
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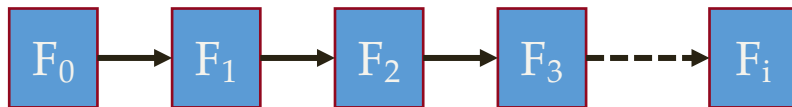


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Basic:  $F_0 = I$   
 Induction:  $F_{i+1} \subseteq \text{Reach}(F_i)$   
 Check:  $F_i \cap \neg P \neq \emptyset$

Unsafety Checking

Backward Sequence  
(over-approximate)

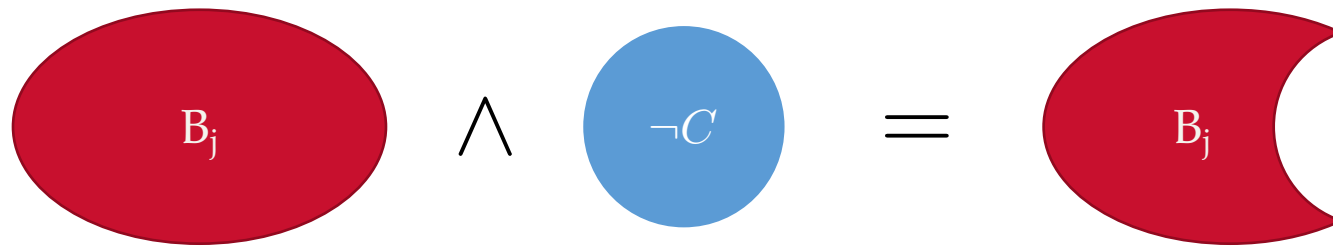


Basic:  $B_0 = \neg P$   
 Induction:  $B_{j+1} \supseteq \text{Reach}^{-1}(B_j)$   
 Terminate:  $B_{j+1} \subseteq \bigcup_{0 \leq k \leq j} B_k$

Safety Checking

# Unsat Cores and CAR

- Unsat cores play a critical role in the performance of CAR
  - Iteratively blocking overapproximate states (B-sequence), much like IC3



- Our quest for smallest unsat cores
  - CARChecker (ICCAD 2017) uses minimal unsat cores – slow!
  - SimpleCAR (CAV 2018) uses first unsat core– fast, but slow convergence
- Tradeoff – smaller v/s faster
  - Find smaller (not minimal) unsat cores fast
- We propose heuristics that find smaller cores; negligible overhead

# Assumptions and SAT Solver

$$\text{SAT}(\varphi, A) \equiv \text{SAT}(\varphi \wedge A)$$

$\varphi$  = Boolean formula in CNF

$A$  = Set of assumption literals

- Query UNSAT  $\rightarrow$  Core  $C \subseteq A$  and  $\varphi \wedge C$  is UNSAT
- $C$  is not necessarily minimal
- Assumption literals are stored in a vector (e.g., MiniSAT)

Let  $A = \{a_0, a_1, a_2, a_3, a_4, a_5, \dots, a_n\}$



- Solver propagates each literal one-by-one; left  $\rightarrow$  right

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- Solver propagates each literal one-by-one; left  $\rightarrow$  right
- Front literals have higher chance to be in unsat core  $C$



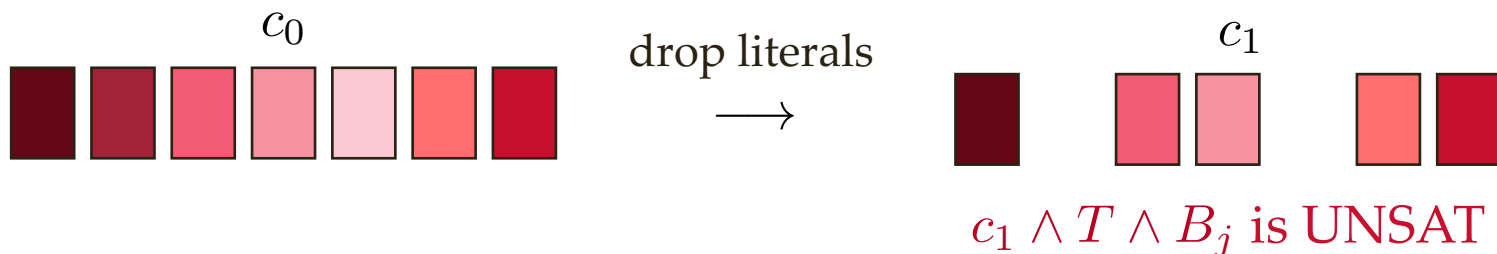
# Proposed Heuristics

- Carefully reorder the assumption literals
  - Drives SAT solvers to return smaller unsat cores
- Intuition
  - Use **old** unsat cores to drive search for **new** unsat cores

## Blocking Step

For some state  $s$ , if  $\text{SAT}(T \wedge B_j, s)$  is UNSAT, add  $c \subseteq s$  to  $B_{j+1}$

Let  $\neg c_0$  be the last-added clause to  $B_{j+1}$   $\leftarrow c_0 \wedge T \wedge B_j$  is UNSAT  
(some state  $s$ )



$c_1$  is weaker than  $c_0$ , and blocks more states at  $B_{j+1}$

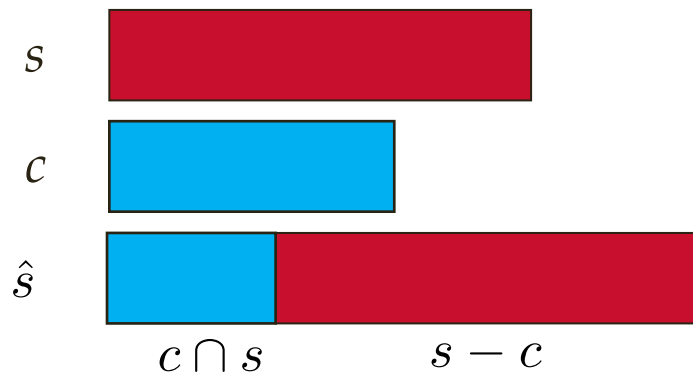
# Heuristic I - Intersection

- **Default:** Let  $s$  be a state to be blocked at  $B_{j+1}$  ( $s$  picked from F-sequence)

Check  $\text{SAT}(T \wedge B_j, s)$

- **Heuristic:** Reorder literals in  $s$  to generate  $\hat{s}$

Let  $\neg c$  be the last clause added to  $B_{j+1}$



Check  $\text{SAT}(T \wedge B_j, \hat{s})$   
(note  $\hat{s} = s$ )

- If UNSAT, higher chance of literals included in unsat core
- Weaker clause; more states than  $c \cap s$  blocked at  $B_{j+1}$



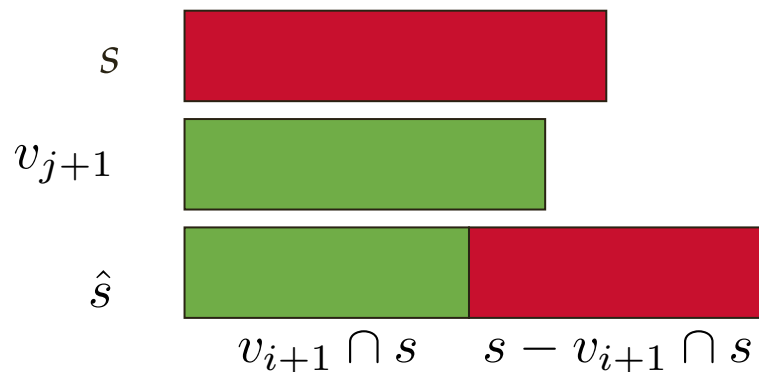
# Heuristic II - Rotation

- CAR picks state from the F-sequence; checks intersection with bad states
  - Ideally, want states to explore disjoint parts of the state space
- Default:** Let  $s$  be a state to be blocked at  $B_{j+1}$  ( $s$  picked from F-sequence)

Check  $\text{SAT}(T \wedge B_j, s)$

If SAT, the assignment is a state  $t$ ; can be reached from  $s$ . State  $t$  is added to F-sequence

- A set of states  $S$  is *diverse* if  $\bigcap_{t \in S} t = \emptyset$ ; disjoint states
- Heuristic:** Reorder literals in  $s$  to generate
  - Every  $B_j$  ( $j > 0$ ) is associated with  $v_j$  to store assumptions from last  $B_{j-1}$  query



Check  $\text{SAT}(T \wedge B_j, \hat{s})$   
(note  $\hat{s} = s$ )

- Generate diverse states whenever query is SAT (proof in the paper)

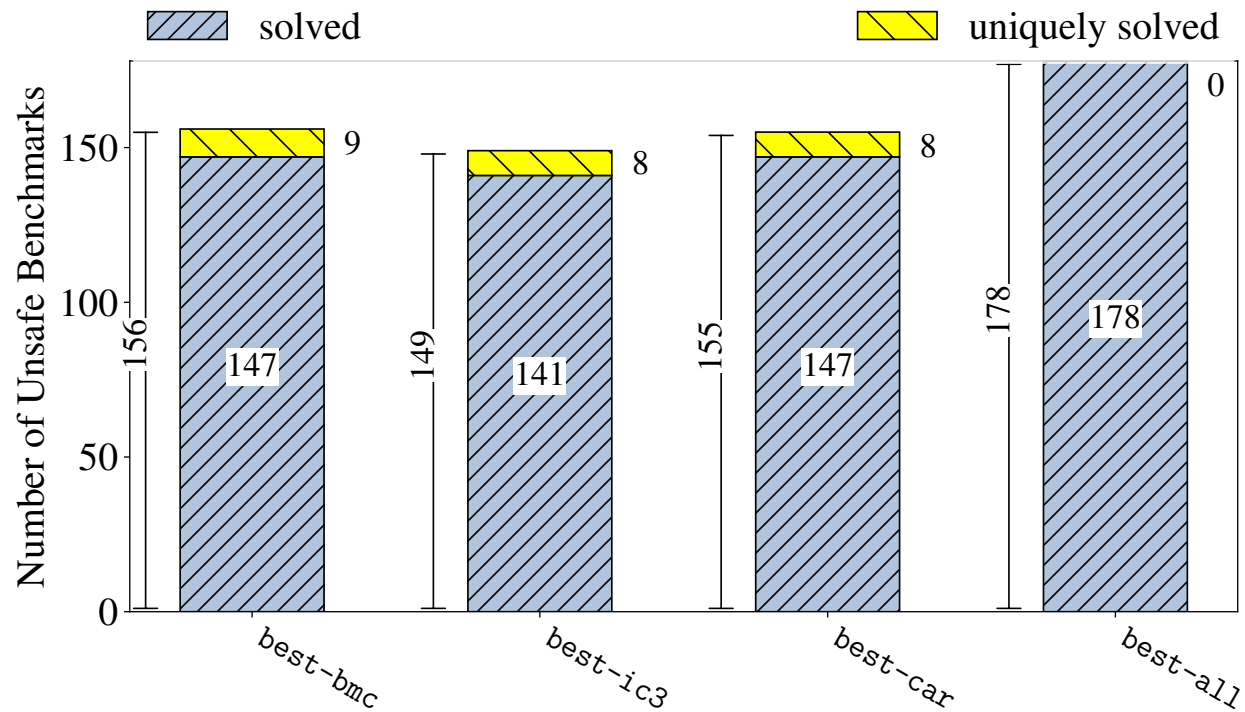
# Experimental Evaluation

- Extended SimpleCAR to include proposed heuristics
  - Intersection, Rotation, Combination, or None
  - Order of state enumeration; pick  $s$  from F-sequence
- Tools and algorithm categories compared:
  - ABC (pdr, 3 x bmc)
  - IIMC (bmc, ic3, quip, ic3r)
  - IC3Ref (ic3)
  - SimpliC3 (bmc, 3 x ic3, Avy)
  - SimpleCAR (8 x car)
- 5 tools, 22 algorithms, 748 SINGLE property benchmarks from HWMCC
- 1 hour timeout
- Identified a bug, and counterexample generation errors
- **We focus on unsafety checking**

Open-source under GNU GPLv3  
<http://temporallogic.org/research/VSTTE19/>

# High-level Performance

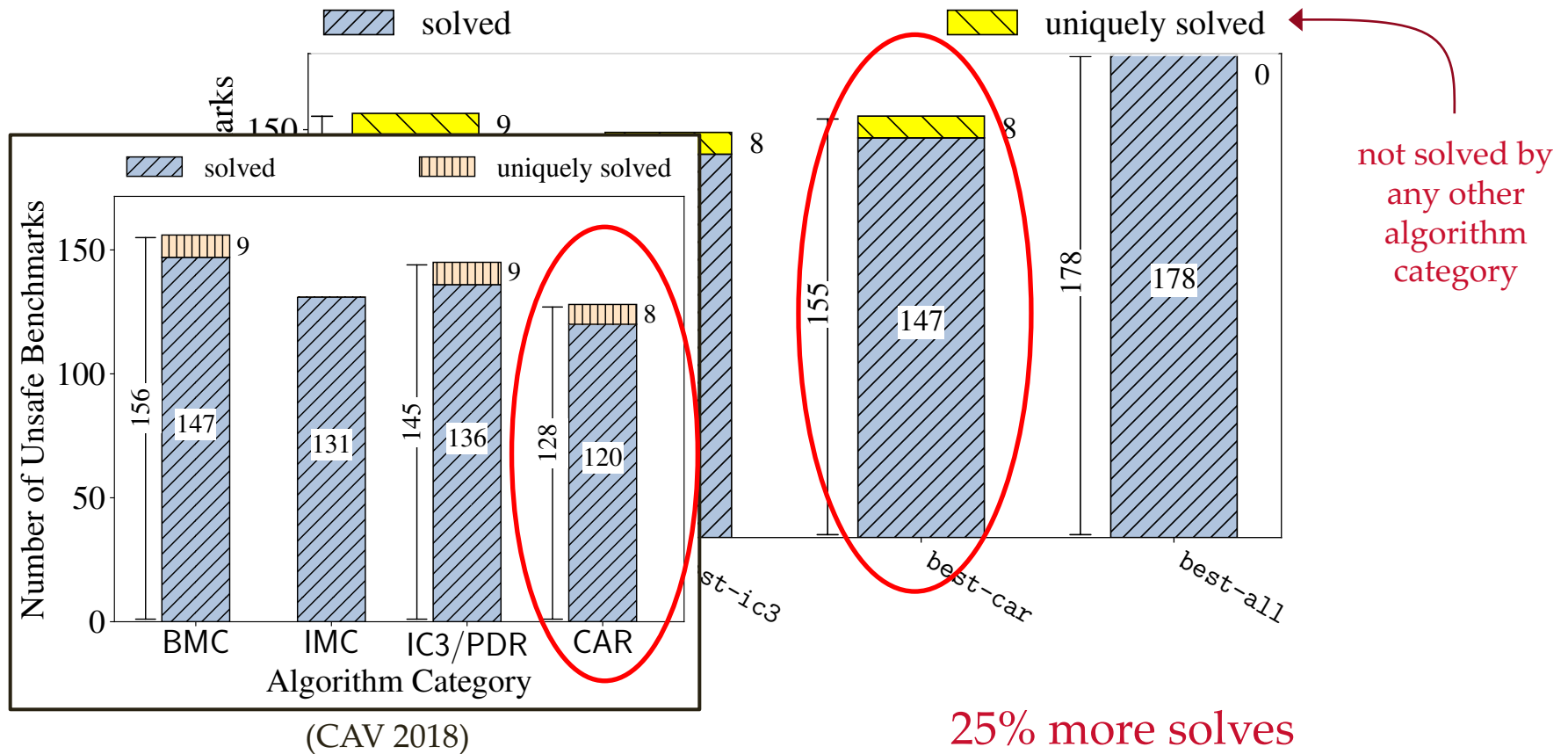
## Algorithm Categories



not solved by  
any other  
algorithm  
category

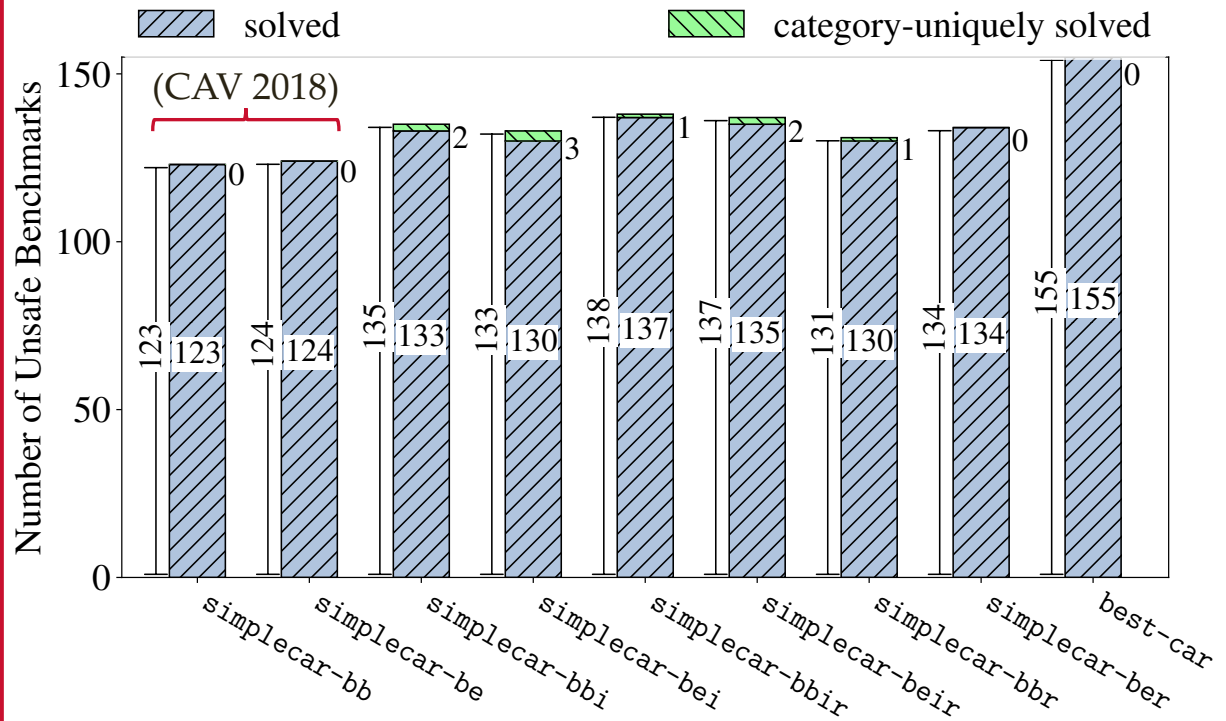
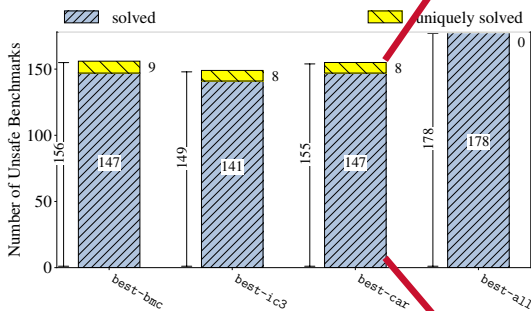
# High-level Performance

## Algorithm Categories



# High-level Performance

## Virtual-best CAR



simpicar-bbir gives 20%  
smaller unsat cores

On-average 30% faster

**Faster convergence!**

# Summary and Discussion

- Design-space exploration via model checking; many models/requirements
- Focus along four verticals
  - Design-space reduction
  - Incremental verification
  - Improved orchestration
  - Model checking algorithms
- Applicable to equivalence checking, product lines, regression runs, etc.
  - Extensions to existing algorithms, and new specialized algorithms
- Better handling of SAT queries improves model checking performance
  - Proposed two heuristics: Intersection and Rotation
- Heuristics can also be applied for clause generalization in IC3
- Future work and research questions
  - SAT-solver internal heuristics for literal scoring
  - Adapting CAR to handle multiple properties; clause sharing between properties
  - Improved synergy between model checking algorithms and SAT solvers

Thank You!

<http://temporallogic.org/research/VSTTE19/>